Projectile motion (ground to ground)

Objective : Theory of projectile motion and application of relations obtained (R, H, TOF, eq of path)

Fill in the blanks

For the following question consider initial velocity of the projectile to be u and angle of projection w.r.t. the ground as θ.

1. When a body is projected at a speed u at an angle θ w.r.t. the ground then horizontal and vertical components of its initial velocity are given by _____ and _____.
2. Horizontal component of velocity of a projectile is given by the relation _____
3. Vertical component of velocity of a projectile is given by the relation _____
4. Horizontal component of velocity of a projectile, at the highest point of its trajectory is _____
5. Vertical component of velocity of a projectile, at the highest point of its trajectory is _____
6. Instantaneous velocity of a projectile, at the highest point of its trajectory is _____
7. Time of a ascent of a projectile is given by TOA = _____.
8. Maximum height reached by a projectile is given by H = _____.
9. Horizontal displacement of a projectile is given by the relation x = _____.
10. Vertical displacement of a projectile is given by the relation y = _____.
11. Range of a projectile is given by R = _____.
12. Range of a projectile is the maximum when angle of projection is ___ degrees or ____ radians.
13. The coordinates (x, y) of a projectile are related by _____.
14. Acceleration of a projectile, in the component form, is given by the relation $\boldsymbol{g}$ = _____.

True / False

1. Keeping the speed of projection constant, horizontal component of initial velocity of projectile decreases with increase in angle of projection from 0° to 90°. (True / False)
2. Keeping the speed of projection constant, time of flight the projectile increases with increase in angle of projection from 0° to 90°. (True / False)
3. Keeping the speed of projection constant, range of the projectile increases with increase in angle of projection from 0° to 90°. (True / False)
4. For a fixed initial speed, a projectile can never attain the same range (R) for two different angles of projection. (i.e. between 0° and 90°) (True / False)
5. For a fixed speed of projection, a projectile can never attain same height (H) for two different angles of projection. (i.e. between 0° and 90°) (True / False)

6. Velocity of a projectile is zero at the highest point of it path. (True / False)
7. Acceleration of the projectile is the maximum at the highest point of its path. (True / False)
8. Average velocity of a projectile for its complete motion is $u\cos(\theta)$. (True / False)

Numericals

1. A body is projected at a speed of 10 ms^{-1} at an angle 30° w.r.t. the ground. Find (i) TOF (ii) Range (iii) Height (iv) write the equation for the path followed by it.
2. A body is projected at a speed of 20ms^{-1} at two angles 30° and θ such that its range is same.
   (i) What is the other angle of projection (θ)?
   (ii) What is the ratio of time of flights for these angles of projection?
   (iii) What is the ratio of heights for these angles of projection?
3. Initial velocity of a body is given by the relation $\bar{u} = 20\hat{i} + 20\sqrt{3}\,\hat{j}$. Find (i) magnitude of initial velocity (ii) angle of projection (iii) TOF (iv) range (v) height.
4. Plot the following graphs in case of a projectile (in each case you may assume the other parameters to be constant)
   (a) Horizontal component of velocity as a function of time
   (b) Vertical component of velocity as a function of time
   (c) Horizontal displacement as a function of time
   (d) Vertical displacement as a function of time
   (e) Horizontal component of acceleration as a function of time
   (f) Vertical component of acceleration as a function of time
   (g) Maximum height reached as a function of initial velocity
   (h) TOA as a function of initial velocity

Answers

Fill in the blanks

1. $u\cos(\theta)$ and $u\sin(\theta)$
2. $v_x = u\cos(\theta)$
3. $v_y = u\sin(\theta) - gt$
4. $u\cos(\theta)$
5. zero
6. $u\cos(\theta)$
7. $u\sin(\theta)/g$
8. $u^2\sin^2(\theta)/2g$
9. $u\cos(\theta) \times t$
10. $u\sin(\theta)\, t - \frac{1}{2} gt^2$
11. $u^2\sin(2\theta)/g$
12. 45°, $\pi/4$
13. $y = x\tan(\theta) - gx^2 / 2u^2\cos^2(\theta)$
14. $\boldsymbol{g} = 0\hat{i} - 9.8\hat{j}$

True / False

1. T
2. T
3. F
4. F
5. T
6. F
7. F
8. T

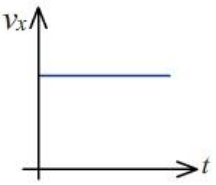
Numericals

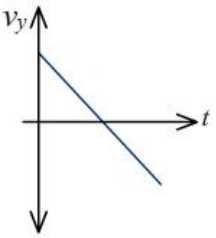
1. (i) 1 s (ii) $5\sqrt{3}$ m (iii) 5/4 m (iv) $y = x/\sqrt{3} - x^2/15$

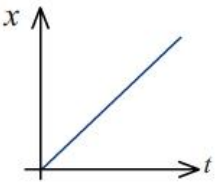
2. (i) 60° (ii) $1/\sqrt{3}$ (iii) 1:3

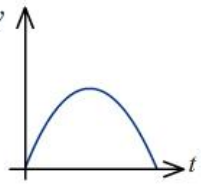
3. (i) 40 ms^{-1} (ii) 60° (iii) $4\sqrt{3}$ s (iv) $80\sqrt{3}$ m (v) 60 m

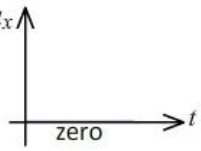
4.

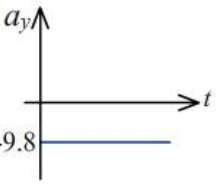
(a) 

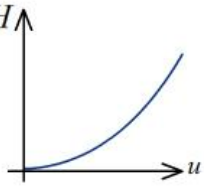
(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

(h)